Coordinate Transform Definition

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*Abstract*— In order to generate a transformation matrix efficiently, a method is devised using three angular parameters and three translation parameters. The method simply combines an angular parameter with a unit vector generated by using standard spherical coordinates which generates a description of axis-angle rotation. This is then concatenated with a translation vector and can be easily applied to a series of augmented vectors.

# Spherical Unit Vector Description

The projector model is made up with 5 main variables. These variables describe the input ray, the two galvanometer axes and the two mirror normal vectors. Thus, all five variables have a unit vector contained within them. To generate these unit vectors without constraints on the magnitude or further normalization calculations, spherical coordinates are used.

|  |  |
| --- | --- |
|  | (1) |

# Quaternion Description

In general, an axis angle rotation can be described with unit vector   combined with magnitude of rotation by using quaternions. Since can be described in spherical coordinates with the radius the quaternion can be written as a function of 3 angles .

|  |  |
| --- | --- |
|  | (2) |

# Rotation matrix description

Ultimately, the quaternion is used to generate a rotation matrix.

|  |  |
| --- | --- |
|  | (3) |

The rotation can be applied to vector such that is the product of the rotation matrix and original vector . When writing the vectors as row vectors the rotation matrix which is a 3 3 must be applied via right multiplication. Note that most standard rotation matrices are left multiplied to a column vector.

|  |  |
| --- | --- |
|  | (4) |
|  | (5) |
|  | (6) |

# Transformation Matrix

A translation described by a row vector can be added to the rotation matrix to describe a new transformed point. We will denote the new transformed vector which also combines translation.

|  |  |
| --- | --- |
|  | (7) |

However, if an augmented vector is used in the transform we can write a new matrix when multiplied to generates .

|  |  |
| --- | --- |
|  | (8) |

We can check that matrix multiplication is still well defined since is 1 4 and is 4 3.

|  |  |
| --- | --- |
|  | (9) |

The reason that this notation is used is that we can augment multiple rows each containing a vector and apply the same transformation in a single matrix multiplication.

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |
|  | (12) |

To check the dimensionality we see that **P** is a 4 matrix and is a 4 3 matrix. Therefore, is 3 as expected.